L4.10

Deutsch Algorithm

find out if a coin is fair or fake

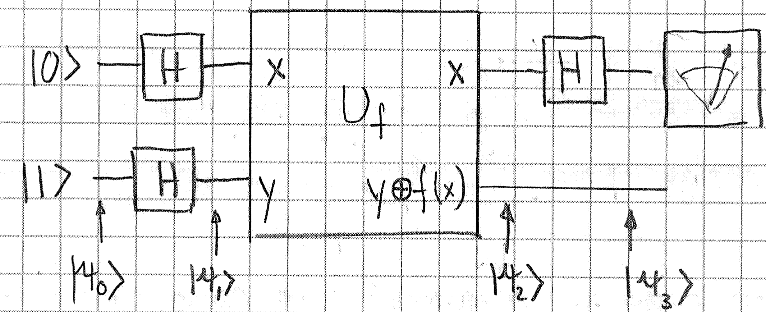
find out if a function is constant or balanced (equal times 0 and 1)

$f : \{\text{front}, \text{back}\} \longrightarrow \{\text{head}, \text{tail}\}$

$f : \{0, 1\} \longrightarrow \{0, 1\}$

$f(0)$	$f(1)$		
0	0	const	$f(0) = f(1)$
0	1	$f(x) = x$, balanced	$f(0) \neq f(1)$
1	0	$f(x) = 1 - x$, balanced	$f(0) \neq f(1)$
1	1	const	$f(0) = f(1)$

(see Nielsen, Chuang pp 28 - 34)



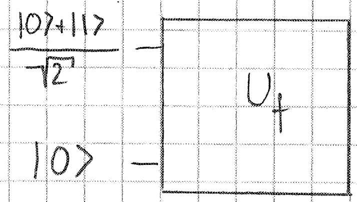
$$U_f : |x, y\rangle \longrightarrow |x, y \oplus f(x)\rangle$$

$\oplus$ is the addition modulo 2

$$U_{f(x)=0} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad U_{f(x)=1} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$U_{f(x)=x} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\text{CNOT}} \quad U_{f(x)=1-x} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

quantum parallism



$$\frac{|0, f(0)\rangle + |1, f(1)\rangle}{\sqrt{2}}$$

superposition of results

L4.12

$$|\psi_0\rangle = |0\rangle|1\rangle$$

$$|\psi_1\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}\frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

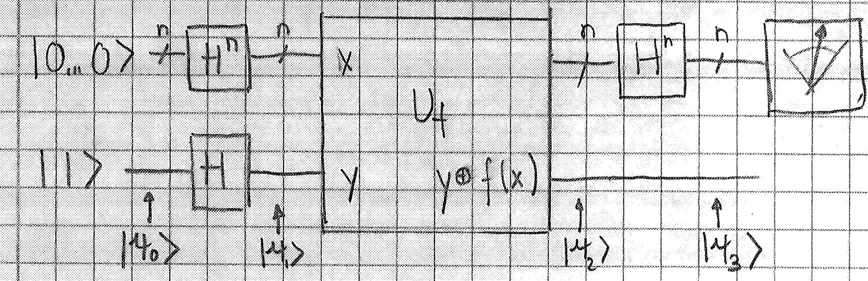
const	balanced
$f(0)=f(1)$	$f(0)\neq f(1)$
$\lvert\psi_2\rangle = \pm\frac{\lvert 0\rangle+\lvert 1\rangle}{\sqrt{2}}\frac{\lvert 0\rangle-\lvert 1\rangle}{\sqrt{2}}$	$\lvert\psi_2\rangle = \pm\frac{\lvert 0\rangle-\lvert 1\rangle}{\sqrt{2}}\frac{\lvert 0\rangle-\lvert 1\rangle}{\sqrt{2}}$

note

$$|x\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}} \xrightarrow{U_f} (-1)^{f(x)}|x\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

$\lvert\psi_3\rangle = \pm\lvert 0\rangle\frac{\lvert 0\rangle-\lvert 1\rangle}{\sqrt{2}}$	$\lvert\psi_3\rangle = \pm\lvert 1\rangle\frac{\lvert 0\rangle-\lvert 1\rangle}{\sqrt{2}}$

L4.13 Deutsch Josza



$$f : \{0\ldots00,\ 0\ldots01,\ \ldots\ 1\ldots11\} \longrightarrow \{0,1\}$$

$$H^n = H \otimes H \otimes \ldots \otimes H$$

example $H^2 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$

$$|\psi_1\rangle = \frac{|0\ldots00\rangle + |0\ldots01\rangle + \ldots\ |1\ldots11\rangle}{\sqrt{2^n}} \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$= \sum_{x \in \{0,1\}^n} \frac{|x\rangle}{\sqrt{2^n}} \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$|\psi_2\rangle = \sum_{x \in \{0,1\}^n} \frac{(-1)^{f(x)}\,|x\rangle}{\sqrt{2^n}} \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

(see Nielsen Chuang pp 34-36)

L5.1

Grover algorithm (Nielsen Chuang, chapter 6)

search in unsorted database

e.g search for number telephonebook

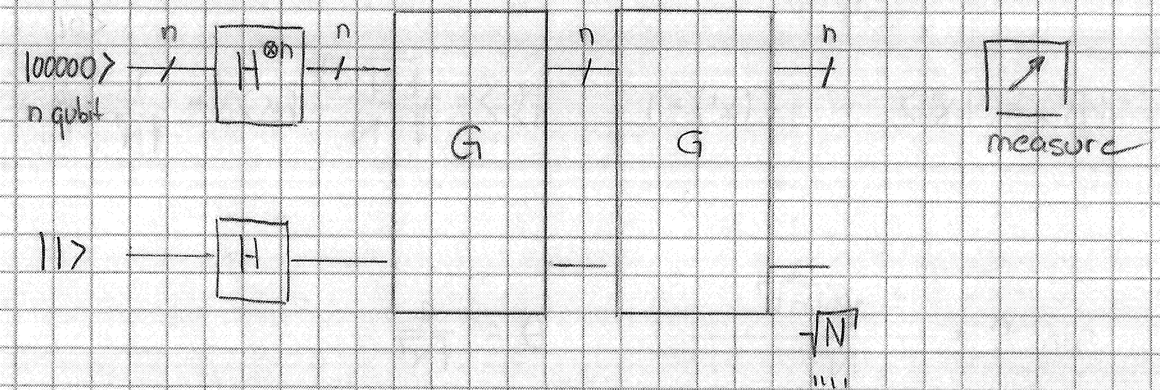
$f: \{\text{index}\} \longrightarrow \{\text{not found, found}\}$

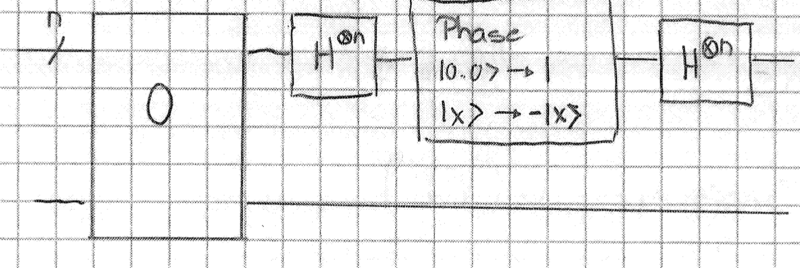
$f: \{0..00,\ 0..01, .. 111..\} \longrightarrow \{0, 1\}$

$N = 2^n$

$$|x\rangle|y\rangle \xrightarrow{\ O\ } |x\rangle|y \oplus f(x)\rangle$$

$$|x\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \xrightarrow{\ O\ } (-1)^{f(x)} |x\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$





$$Ph: \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & \ddots \end{bmatrix} = 2|0\rangle\langle 0| - \mathbb{1}$$

$$H^{\otimes n}\left(2|0\rangle\langle 0| - \mathbb{1}\right) H^{\otimes n} = 2|\xi\rangle\langle\xi| - \mathbb{1}$$

$$|\xi\rangle = H^{\otimes n}|0\rangle = \frac{1}{\sqrt{2^n}} \sum |x\rangle = \frac{1}{\sqrt{2^n}}\left(|0..0\rangle + |0...01\rangle \ \ldots\ |1...1\rangle\right)$$

$2|\xi\rangle\langle\xi| - \mathbb{1}$

$$\sum_k \alpha_k |k\rangle \longrightarrow \sum_k \left(2\langle\alpha\rangle - \alpha_k\right)|k\rangle \quad \text{where} \quad \langle\alpha\rangle = \frac{1}{N}\sum_k \alpha_k$$

rotation about the mean

$$G = \left(2|\xi\rangle\langle\xi| - \mathbb{1}\right) O$$

geometric visualization

$$|\alpha\rangle = \frac{1}{\sqrt{N-1}} \sum_{\substack{x \\ f(x)=0}} |x\rangle$$

$$|\beta\rangle = |x'\rangle \qquad f(x') = 1 \qquad |\xi\rangle = \sqrt{\frac{N-1}{N}}\,|\alpha\rangle + \frac{1}{\sqrt{N}}|\beta\rangle$$

$$\sin\theta = \frac{2\sqrt{N-1}}{N} \qquad \theta \approx \frac{2}{\sqrt{N}}$$

$\sqrt{N}$ iterations

quadratic speedup $O(\sqrt{N})$

probablistic

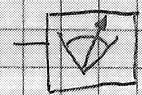
optimal

L4.14

Di Vincenzo Criteria

- Well defined qubits, scalable
- Initialization to a pure state
  $|00..0\rangle$
- Universal set of quantum gates

- Qubit-specific measurement
  with high efficiency



- Long coherence time